Unit - 1

Advanced Abstract Algebra

1. Advanced Abstract Algebra

Group Theory : Normal and Subnormal Series, Composition Series, Lemma of Zasenhaus, Scheier's theorem, Jordon-Holder theorem, Solvable groups and Nilpotent groups.

Factorisation in Integral Domains : Divisibility, Associates, Reducible and Irreducible elements, Prime elements, Greatest common divisor and Lowest common multiple in Euclidean Domains and Principal Ideal domains, Maximal Ideal in P.I.D. and Euclidean domain, Uniques Factorisation Theorem in P.I.D. and Euclidean Domain.

Modules: The concepts of Modules and sub-modules, cyclic modules and sub modules, finitely generated modules, sum and direct sum of sub-modules, Fundamental structure theorem on finitely generated modules over Euclidean rings and Principal ideal domains, Application to finitely generated abelian groups. Quotient modules and linear mappings (homomorphism, endomorphism), Schur's lemma.

Field Theory : Extension field, finite extension, Algebraic elements and minimal polynomial, Algebraic extension of a field and field algebraic elements, simple extensions, Roots of polynomials and multiple roots, Splitting field, separable extensions.

Galois Theory: Group of automorphism of a field and fixed field of a group of automorphism, G(K/F), O[G(K/F)] < [K:F] for a finite extension K of F, Normal extension, characterisations of normal extension, Galois group of a polynomials, Fundamental theorem of Galois Theory, Cyclotomic polynomials, Solvability by radicals.

2. Linear Algebra Projection mappings on a linear space.

The decomposition of a linear space as a direct sum of two spaces which are the range space and the null space of the projection mapping, Quotient space of a linear space.

Isomorphism theorem,s. linear functional, Dualspace and second dual of a linear space. Canonical embedding of a space into its second dual and their isomorphism in the finite dimensional case.

Metric Vector Space: Bilinear form, Matrix representation, sylvester's theorem, Quadratic forms and their reduction to normal form & positive definitive form, Rank and signature, Hermitian forms, Euclidean vector space and Pythagoras theorem, Ortho-normal basis for a finite dimensional Euclidean vector space.

Lattice Theory: The concept of a lattice as an order structure and as algebraic structure and their equivalence. Principle of duality, Distributive lattice. Intervals in a lattice and projection operators associated with an interval. Modular lattices and their characterizations, Complete and complemented lattices. Lattice homomorphism, Fixed point fo order homomorphism of a complete lattice.

Boolean Algebra: Boolean algebra as a complemented distributive lattice and converse. Boolean rings and identification of Boolean algebra and Boolean rings. Sub algebra and generators. Boolean homomorphism and rings homomorphism. Ideal in a Boolean algebra and dual ideal. Fundamental theorem of homomorphism. Stone's representation theorem for Boolean algebra and Boolean Rings, Applications to electrical network and logical puzzles.

Unit - 2

Real Analysis

1. Bolzano Weierstrass theorem, Heine-Borel theorem and Cantor's decreasing set theorem of real analysis. Functions of bounded variation, total variation; Sum, Difference and product of functions of bounded variation, Additive property of total variation, Total variation o n [a, x] as a function of x, functions of bounded variation expressed as difference of increasing functions, continuous functions of bounded variation, Absolutely continuous functions, Implication relationship between absolute continuity, Continuity and bounded variation, Absolute continuity and uniform Lipshitz condition, Absolute continuity of sum, difference, Product and quotient of absolutely continuous functions

3)

Riemann Stielties Integral: Definition; Linearity properties, integration by parts, Reduction to a Riemann Integral, step function as integrators, Conditions for R-S- integrability, Order preservation properties of integrals, Integral as a limit of sum, Mean Value Theorems, Integration and differentiation, Integrators of bounded variation change of variables, Integration of Vector valued functions and Rectifiable curves.

Multi-variable Differential Calculus: Directional derivative and continuity of functions from R^n to R^m Linear functions, Total derivative, The matrix of a Linear function, Jacobian Matrix, mean value theorem for differentiable functions, Sufficient condition for differentiability, Sufficient condition for equality of mixed partial derivatives, Taylor's formula for functions from R^n to R^m

Power Series and Extremum Value Power Series, Radius of convergence, Uniform convergence of Power Series, Abel's theorem, Tauber's theorem, Extremum values for the functions of two and three variables, Extremum problems with constraints, Lagrange's Multiplier Method

Inverse function theorem, Implicit function theorem, Definition of Jacobian, Jacovian's of implicit functions, Necessary and sufficient condition for a Jacobian to vanish co-variants and invariants

2. Lebesgue Measurable Sets : Lebesgue outer measure and its properties, Lebesgue measure and Lebesgue measurable sets, Necessary and sufficient conditin for a set to be measurable, Set operations on Lebesgue measurable sets, sequence of measurable sets, Total additivity of lebesgue measure, Set ofmeasure zero, Canter's ternary set and its measurability, Brel sets and their measurability.

Lebesgue Measurable Functions : Definitions of Lebesgue measurable functions and their equivalences, properties of measurable functions, Arithmetical and analytical operations on measurable functions, measurability of continuous function, Notions of characteristic function, Simple function, Step function and their measurability, Properties of almost everywhere, Borel measurable functions, Sequence of functions, Egoroff's theorem, Lusin theorem and Frechet theorem.

8

Lebesgue Integration : Lebesgue integral of a simple function, Lebesgue integral of a bounded measurable function, comparison of Reimann and Lebesgue integral of non-negative measurable function, elementary properties of lebesgue integral Additivey, order preservation and integrability properties, Examples of unbounded non-negative measurable functions which are (i) integrable over [0, 1] and (ii) which are integrable over [0, 1].

Convergence theorems: Convergence in measure, Riesz theorem, Bounded convergence theorem, Fatou's Lemma, Lebesgue monotone convergence theorem, Lebesgue dominated convergence theorem, Vitali's covering theorem and Lebesgue theorem.

Differentiation and Integration: Functions of bounded variation and its properties, variation function of a function of bounded variation, jordan's recomposition theorem, Absolutely continuous functions and their properties, Differentiation of an integral, Fundamentals theorem of integral calculus, Lebesgue Sets and integral derivatives

Unit - 3

Elementary concepts of topological space: Topology with open sets as primitive concept, Intersection of topologies, Closed sets, Neighbourhoods, Adherent point, Accumulation point, Derived sets, Closure, Interior, Boundary, Theorems connecting these elementary concepts in a topological space. Convergence of a sequence, convergence and accumulation points, Derived sets and closure in subspace in terms of corresponding concepts in original space. Concept of basic (Open base) and subbase (Open subbase) of a topological space. Usual topology on R and metric topology.

Continuous mappings between topological spaces, Characterisations of continuity by neighbourhoods, open sets, closed sets and closure, sequential continuity and its relationship with continuity, Homomorphism, product space of a finite family of topological spaces, convergence in a finite product space and convergence coordinate-wise. Product space of an arbitrary family of spaces defining open base and subbase, projection mappings on product-space.

Separation aximons in a topological space. T_{0-}, T_{1-}, T_{2-}, spaces and their mutual implication relationship. Characterization of T_{0-}, T_{1-}, T_{2-}, spaces, unique limit of convergent sequences in a T_{2-}, space. Regular and Normal spaces, T_{2-} and T_{4-} spaces, characterization in terms of open neighbourhood of a point and open neighbourhood of a closed set.Urysohn's lemma and Tietzz Extension theorem.

Compactness, Compact space, Characterisation of compactness in terms of closed sets with finite intersection properties, Compact sets in a topological space, Compactness of closed sets in a compact space. Compact sets in a Housdorfrf space. Normality of a compact Housdorff space, compactness of continuous image of a compact space.

Connectedness: Connected and Disconnected spaces, Characterisations of disconnected space in terms of continuous mappings onto a two point discrete space, Continuous image of a connected space, Connected subsets of a topological space, Connectedness of closure of a connected set, Connectedness of union of a family of connected sets with non empty intersection, connected subsets of R under usual topology.

Unit - 4

Complex Analysis

Analysis Functions, Cauchy: -Riemann differential equations. Sufficient conditions for differentiability, power series, radius of convergence, Hadmard formula, power series represents an analytic function inside its circle of convergence, Abel's theorem on convergence of power series, Angle preserving property of analytic functions.

Bilinerar Transformations: Extended complex plane, Resultant and inverse of bilinear transformations, Cross-ratio preserving property, Preservation of the family of straight lines and circles, Fixed points and normal form of a bilinear transformation. Some special bilinear transformations conformal Mappings: Necessary and sufficient condition for a mapping to be conformal. The mappings $w = z^n$, $w = z^2$ and the inverse mapping $w = z^{1/2}$. The exponential mapping $w = e^z$ and the logarithmic mapping $w = \log^2$. Riemann's Theorem on conformal mapping.

Complex Integration: Complex line integrals, Cauchy's theorem, Cauchy's integral, formula, Poisson's integral formula cauchy's integral formula for derivatives, Cauchy's inequality, Taylor's theorem, Laurent's theorem, Lioville's theorem, Morera's theorem.

Residue Theory: Singulatities of an analytic function, Isolated and essential singularities, Poles and residues, Cauchy's theorem of residues, Evaluation of real integrals with the help of Contour integrals, Principal of Arguments, Rouche's theorem, Fundamental theorem of Algebra, Meromorphic Functions, Rational Functions, Maximum modulus principle.

Applications: The dirichlet Problem: Solution and uniqueness of solution Green's functions, Existence and Uniqueness of Green's functions, Neumann problem and solution conformal mapping, analytic continuation.

Unit - 5

1. Differential Equation

Existence Method: Initial value problem and the equivalent integral equation, concept of local existence, existence in the large and uniqueness of solutions with examples. Existence and Uniqueness of solution of ordinary differential equation of first order, Picards method, Cauchy's Lipshitz method, Picards, iteration method for an approximate solution of the initial value problem, Runge-Kutta's method.

Basic Theorem: Ascoli-Arzela theorem. A theorem on convergence of a family of initial value problem.

Picards – Lindelof theorem – Peano's existence theorem and corollary, Maximal intervals of existence, Extension theorem and corollaries, Kamlee's convergence theorem, Kueser's theorem (statement only).

Differential Inequalities and uniqueness – Gron-wall's inequality, maximal and Minimal solutions, Differential inequalities. A theorem of winter uniqueness theorems. Nagesmo's and Osgood's criteria.

System of Differential Equations: Methods of solving a system of differential equations by reducing in to a single equation, solution by finding integrable combinations. Applications to systems of linear differential equation with constant co-effiient.

Stability: Stability of autonomous system of differential equation. Stationary points of autonomous systems and its classification as stable, asymptotically stable and unstable stability for linear systems with constant coefficients.

Polynomials: Generating function for Legendere Laguerre and Hermite polynomials, their recurrence relations, orthogonality and formula, Rodrgae's Bessel's generating recurrence relation for Bessel's function, orthogonality of Bessel functions.

2. Partial Differential Equation

Partial differential equation of the first order, Cauchy's problem, linear equation, integral surfaces, orthogonal surfaces, Non-linear partial differential equations Cauchy's method of characteristics, charpit's method, Jacobi's method.

Partial differential equation of the second order, linear partial differential equation with constant co-efficient, equations with variable co-efficient, characteristic curves, characteristic equations, separation of variable Non linear equations of the second order.

Classification and solution of partial differential equations reduction of linear partial differential equation in two independent variables to canonical forms and their classification into elliptic, parabolic and hyperbolic forms, laplaces's heat and wave equation in one dimension solution by the method of characteristics

Transport-equations - Initial value problems, non homogeneous equation. Laplaces's equations - fundamental solutions, families of equipotential surfaces, boundary value problems Mean value formula, separation of variables, properties of harmonic function Green's function.

Heat equation - Fundamental solutions, mean value formula properties of solution wave equation - elementary solution of one-dimensional wave equation, Green's solution of wave equation, Green's function of wave equation, Non homogeneous wave equation.

Unit - 6
Analytical Dynamics

Generalised coordinates, holonomic and non-holonomic systems seleronomic and Rhenomic system, generalised potential, lagrange's equation for impulsive motion lagrange's equations of second kind, uniqueness of solution energy equation for conservative fields.
Hamilton's variables Hamilton's canonical equations cyclic coordinates, Routh's equations, variational principle harmilton's principle principle of least action.

Small oscillations of conservative systems, lagrange's equations for small oscillations, principal oscillations and normal oscillations real roots of the lagrangian determinant oscillations under constraint.

Canonical transformations, lagrange's brackets, poissons' brackets Poisson's identity Jacobi-poisson's theorem, conditions of canonocal character of transformation in terms of lagrange's brackets and poisson's brackets, invariance of lagrange's brackets and poisson's brackets under canonical transformations

Hamilton-Jacobi equation, Jacobi's theorem, application of Hamilton - Jacobi theory to solve Kepler's problem and harmonic oscillators, Euler's dynamical equations for the motion of a rigid body, motion of a rigid body about an axis, Motion about revolving axis.

Unit - 7

Set Theory: Countable and uncountable sets. Infinite sets and the Axion of choice. Cardinal numbers and its arithmetic. Schroeder – Bevmstenin theorem, Cantor's theorem and the Continuum hypothesis, Zorn's lemma, Well-ordering theorem.

Graph Theory: Definition of graph, paths, circuits, cycles and sub graphs. Induced sub-graph, degree of a vertex, connectivity planar graphs and their properties and trees.

Tensor Algebra: Contravariant and Covariant transformations, contra variant and covariant tensor, Tensor of the type (r, s), Symmetric and skew symmetric tensor, addition and multiplication of tensor, quotient law, Law of Contraction, linner and outer product, associated Vectors, Coordinate Curves, Coordinate hyper surfaces, Angle between two vectors. Christoffel's symbols, Transformation, Covariant derivative of tensor.

Integral Transforms: Laplace transforms, Definition, Laplace Transformation of t^n, e^{nt} Sinhat, Coshat, Sinat, Cosat, Convolution theorem, Inversion theorem, Application to Linear differential Equation with Constant Coefficient Fourier Transforms, Fourier Integral theorem, Fourier sine and Cosine transforms.

Unit - 8

Numerical Analysis

Method of finite difference: Difference formula, fundamental theorem of the difference calculus, operators E and D and their properties, relations between difference and differential operators, effects of an error in a tabulator value one or more missing terms. Factorial notation and methods of representing any given polynomial in factorial notation, differences difference of zero and recurrence relation between them. Leibnitz's rule.

Solution of Albebraic and Transcendental Equations: Errors and their analysis, general error formula bisection method, false-position method iteration method, Newton-raphson's method, Newton's iterative formula for finding the inverse-square root etc. chebyshev formula comparison of Newton's method with Regular false method. Graeffis root squaring method.

Interpolation: Newton's Formula for interpolation, Gauss's central difference formula, staring formula, lagrange's interpolation formula, divided differences and their properties Newton's general interpolation formula interpolation by iteration, inverse-interpolation.

Differentiation and integration method: Derivatives from interpolation polynomials, Maximum and minimum value of a tabulator function, General quadrature formula for equidistant ordinates Trapezoidal rule, Simpson's 1/3 rule, Simpson's 3/8 rule, weddle's rule.

Difference equations: Homogeneous linear equation with cosntant coefficients, Non-homogeneous linear equation Principle of least square, Fitting of polynomials, change of origin and scale for simplifying the calculations reduction of non-linear equations to linear form.

Unit - 9

Functional Analysis

Banach Space: Norm and normed linear space, Banach space, The Banach spaces R", C" lp,l,C[a, b], Lp" continuity of norm function and linear operations, Quotient space, continuous and bounded linear transformations and normed linear spaces.

The space of all continuous linear transformations on a normed linear space, linear functional dual space second dual isomorphism and embedding a normal linear space in its second dual space reflexive spaces dual space of Rn", lp (1< p <), Co. finite dimensional normed linear spaces and equivalent norms. Lemma of F. Reisz.

The Hahn-Banach Theorem and its consequences, Banach Steinhauss theorem and some of its applications. The open mapping theorem. The closed graph theorem, The concept of projection on Banach spaces and geometric characterisations

Inner product space and Hibert space. Inner product space, Hilbert Space, Examples and counter examples, Cauchy - Schwartz in equality and continuity of inner product parallelogram law polarisation identity. Convex sets and lemma of F. Riesz and closed and convex sets in a Hilbert space, Orthogonal complements pythagorean theorem. The projection theorem on a Hilbert space.

Orthonormal sets in a Hilbert space, Bessel's inequality parseval's relation complete orthonormal sets and characterisation theorem for complete orthonormal sets in a Hilbert space. Frichet-Riesz representation theorem for bounded linear functional in Hilbert space. operator on a Hilbert space and adjoint of an operator. Adjoint operation on B (H) and its properties. Self adjoint operators and its properties.

Unit - 10

Fluid Mechanics

Kinematics of Fluid Motion: The continuum view of fluid mechanics, Velocity at a Point, Stream lines and path lines, Velocity potential, Vorticity Vector, Local and Particle rates of change. Equation of continuity in Cartesian, Polar and Spherical co-ordinaties, Equation of Continuity in vector forms, Boundary surface, Rotational and Irrotational Motion.

Equation of Motion: Euler's equations of motion, Bernoulli's equation, Steady Motion, Equation under Impulsive Force, Motion of a small fluid element, Flow and Circulation, Stoke's theorem, Kelvin's circulation theorem, Kinetic Energy of finite and infinite fluid, Kelvin's minimum Energy theorem.

Motion in Two-Dimension: Stream Function, Complex Potential for Two-deimneional, irrotational Incompressible Flow, complex Potential for line Source, Line Sink and Line Doublets, Two Dimensional Image System, Source Sink and Doublets, Image in a rigid infinite plane, Vortex Motion, Strength of Vortex, Simple Problems in Vortex Motion, Parallel Line Vortices.

Motion of Fluid Past Cylinder and Sphere: Fluid Streaming past a fixed circular Cylinder, Elliptic Cylinder and Sphere, Kinetic Energy of a Rotating Elliptic Cylinder, Motion of Fluid between Rotating Co-oxial circular Cylinder.

Viscous Flow: Rate of strain quadric, Stress analysis in a fluid in motion, Relation between stress and rate of strain, Navier-Stokes equation of motion of a viscous fluid, Solution of Navier - Stokes equation for the incompressible couette flow between two flat parallel plates, Velocity and temperature distribution for simple flow problems.

****